New averaged Reynolds equation based on wall slip boundary conditions

A. Fatu*, M. Hajjam

Institut Pprime, Département Génie Mécanique et Systèmes Complexes, IUT d'Angoulême, 16021 Angoulême Cedex, France

*Corresponding author for tribo-lyon2013@sciencesconf.org

1. Introduction

In the case of relatively small size of contacts, deterministic simulation techniques can be used for the investigation of hydrodynamic lubrication of textured bearings. However, complete deterministic solutions of large bearings are not possible in acceptable amounts of computing time. The solution comes from average flow models, developed initially for hydrodynamic lubrication of rough sliding surfaces. The main idea is to modify the Reynolds equation by the introduction of flow factors computed by deterministic simulations on a small but representative part of the bearing [1, 2]. Several authors shown that the flow factors depend on the inter-asperity cavitation, asperity elastic deformation and thermal effects. In the specific case of textured surfaces, de Kraker et al. [3] proved that convective inertia forces may also play a role in evaluating the flow factors.

2. Model presentation

In our study, a new average Reynolds equation is proposed. The start point is a parallel between the influences of textured and wall slip in hydrodynamic lubrication. It is suppose here that the flow between textured surfaces is equivalent with the flow between smooth surfaces with wall slip boundary conditions.

When slip is possible,the wall velocity is proportional to the shear stress, with proportionality factors referred as α_1 for the lower surface and α_2 for the upper surface. If the slip length is defined as $l_s = \alpha\mu$, with μ the fluid dynamic viscosity, the isoviscous 2-D Reynolds equation, that assumes a laminar flow with the inertial effects neglected in the filmis:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\mu}\frac{h^2+4h(l_{s1}+l_{s2})+12l_{s1}l_{s2}}{h(h+l_{s1}+l_{s2})}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\mu}\frac{h^2+4h(l_{s1}+l_{s2})+12l_{s1}l_{s2}}{h(h+l_{s1}+l_{s2})}\frac{\partial p}{\partial z}\right)-\frac{h}{2\mu}\frac{\partial p}{\partial x}\frac{\partial h}{\partial x}\frac{hl_{s2}+2l_{s1}l_{s2}}{h+l_{s1}+l_{s2}}-\frac{h}{2\mu}\frac{\partial p}{\partial z}\frac{\partial h}{\partial z}\frac{hl_{s2}+2l_{s1}l_{s2}}{h+l_{s1}+l_{s2}}=\frac{U}{2}\frac{\partial h}{\partial x}+\frac{U}{2}\frac{\partial}{\partial x}\left(h\frac{l_{s2}-l_{s1}}{h+l_{s1}+l_{s2}}\right)-U\frac{\partial h}{\partial x}\frac{l_{s2}}{h+l_{s1}+l_{s2}}+\frac{\partial h}{\partial t} \tag{1}$$

The approximation of the slip lengths is made by using the same technics used to compute the flow factors: by comparing the flow deterministically computed in pure Poiseuille or pure sliding between a textured and a smooth surface with the flow between a smooth surface with wall slip conditions and a second without wall slip conditions. This way, the slip length of lower and upper surface can be computed separately. However, the computation results show that the equivalent slip lengths are different in Poiseuille and pure sliding. Moreover, if the texture is not isotropic, the slip lengths are different in x and z directions. Therefore, equivalent Poiseuille slip lengths are computed in $x(l_{sx})$ and $z(l_{sz})$ directions and equivalent shear slip lengths are estimated for pure sliding conditions (l_s^c). Consequently, eq. (1) becomes:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\mu}\frac{h^2+4h(l_{sx1}+l_{sx2})+12l_{sx1}l_{sx2}}{h(h+l_{sx1}+l_{sx2})}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\mu}\frac{h^2+4h(l_{sz1}+l_{sz2})+12l_{sz1}l_{sz2}}{h(h+l_{sz1}+l_{sz2})}\frac{\partial p}{\partial z}\right)-\frac{h}{2\mu}\frac{\partial p}{\partial x}\frac{\partial h}{\partial x}\frac{hl_{sx2}+2l_{sx1}l_{sx2}}{h+l_{sx1}+l_{sx2}}-\frac{h}{2\mu}\frac{\partial p}{\partial z}\frac{\partial h}{\partial z}\frac{hl_{sz2}+2l_{sz1}l_{sz2}}{h+l_{sz1}+l_{sz2}}=\frac{U}{2}\frac{\partial h}{\partial x}+\frac{U}{2}\frac{\partial}{\partial x}\left(h\frac{l_{s2}^c-l_{s1}^c}{h+l_{s1}^c+l_{s2}^c}\right)-U\frac{\partial h}{\partial x}\frac{l_{s2}^c}{h+l_{s1}^c+l_{s2}^c}+\frac{\partial h}{\partial t} \tag{2}$$

By comparing eq. (2) with the average Reynolds equation proposed in reference [1], several similitudes can be observed. For example it can be written that $\frac{h^2+4h(l_{sx1}+l_{sx2})+12l_{sx1}l_{sx2}}{h(h+l_{sx1}+l_{sx2})}\approx\phi_x$ and $\frac{\partial}{\partial x}\left(h\frac{l_{s2}^c-l_{s1}^c}{h+l_{s1}^c+l_{s2}^c}\right)\approx\sigma\frac{\partial\phi_s}{\partial x}$. However, there are two terms in the left hand side of eq. (2) and one on the right hand side that depends on the h and p derivativesand do not find equivalent in the average flow model.

3. Results

Several simulations have been performed in order to compare results obtained by deterministic calculations with those obtained with the new average Reynolds equation proposed here and with the flow factor model. It is proved that in the case of parallel bearing surfaces, both average models lead to identical predictions. The comparison with the deterministic calculations proves that the computation of the equivalent shear slip length must take into account micro-cavitation phenomenon.

The investigationis next extended to non-parallel bearing surfaces. In comparison with the flow factor model,the new average Reynolds equation shows a better comparison with the deterministic predictions.

4. References

[1] Patir N., and Cheng H.S., “An average flow model for determining effects of three-dimensional roughness on partial hydrodynamic lubrication,” ASME J. Lub. Tech., 100, 1978,12-17.

[2] Bayada G., and Chambat M., “New models in the theory of the hydrodynamic lubrication of rough surfaces,” Trans. ASME, 1988, 110, 402-407.

[3] De Kreker A., van Ostayen R.A.J., Rixen D.J., "Development of a texture averaged Reynolds equation," Tribology International, 100, 2010, 2100-2109.