New mass conserving cavitation algorithm originating from compressible fluid models

G.Bayada*

Institut Camille Jordan ,UMR CNRS 5208, Université de LyonMathématiques -INSA de Lyon
F 69621 Villeurbanne Cedex

*gbayada@insa-lyon.fr

1. Introduction

Most of the mass-conserving cavitation algorithms are based upon the Jakobsson-Floberg-Olsson (JFO)/Elrod-Adams (EA) models[1]. These models are associated with the relation (1) between the pressure and ,the "local proportion "offluidθ:

$$p>=p_c 0<\theta=<1 \; (p-p_c)(1-\theta)=0 \quad (1)$$

The present presentation will address three aspects of these models:
(a)It is difficult to obtain them in a rigorous way using a thin film procedure, starting from a full 3 dimensional description. For example E.A. model is obtained by as a modification of a mass flow description which is however only valid if homogeneous 3-dimensional flow is assumed.
b)All these models assume the fact the pressure never falls below the cavitation pressure p_c. However sub-ambient pressure loop have been observed as early as 1982. The existence of such under-pressure can be neglected for heavily loaded devices. For light loaded devices however, the constraint p>pc cannot be retained and previous models are not convenient. Moreover, variation of the viscosity or the densityinside the cavitation area cannot be taken into account.
(c) The computation of the solution is not easy. Elrod and Adams identify as a difficulty the fact that the relation θ-p in equation (1) is not one to one (for example the value θ=1 is not associated to an unique value of the pressure) and so modify this relation in the non cavitated area by introducing a small compressibility parameter. Numerous methods have been proposed to deal with both E.A and JFO models. Most of them are based on the E.A algorithm and itsVijayaraghvan –Keith improvement by introducing various iterative process coupling pressure and saturation. Recently it has been proposed to introduce Linear Complementarity numerical method to solve the discretized problem. All these methods are not easy to implement. This explain that despite some physical disturbing feature(it is not mass conserving), Christopherson[2] method is still often used even today .

2. Compressible thin film model

It has been recently proposed to study compressible3-dimensional Navier Stokes equation with variable density and (dynamic) viscosity and rigorously perform a thin film procedure. Assuming simplified properties (barotropic and isentropic assumptions) of the fluid, a Reynolds compressible equation is obtained which is very close to the E.A. model and improved it in some aspects:
-No geometrical assumption about the shape of the cavitation area
-Variation of both density and viscosity in both cavitation region and in "full film" region can be considered.
-No constraint like $p>=p_c$ are introduced so that under-pressure can be obtained.
Detailed properties of this new model which will be called 'fully compressible model"(FC) have been already given [3].

3. New algorithms

We will discuss numerical algorithms tosolve this new model (FC) and it turn show how it can be adapted to solve JFO/EA classical model(1).
The problem is still a non-linear one, involving pressure or density as primary unknown. A fully explicit schema will be first presented. However convergence needs a great number of nodes.Next, an (half) implicit schema will be introduced which in turn can be relatively easily solved for usual compressibility lawsin both cavitation and non-cavitation areas. Stability is very good and the number of nodes required is much smaller than for the (fully) explicit schema.
As the usual JFO/EA model can be viewed as an approximation of the (FC) model, a simplified version of the previous algorithm is proposed to solve problems like problem(1). This version allows the implicit discrete problem to be solved explicitly.This algorithm is very close to the well-known Christopherson algorithm and so is very easy to implement both for steady-state than dynamic problems.

4. References

[1] Elrod, H.G. and Adams, L.,"A Computer program for Cavitation and Starvation*," in Cavitation and related phenomena in lubrication*, Mech. Eng. Pub. for the Institute of Tribology, The University of Leeds, 1975

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