A 3D Linear Elastic Multigrid Model for Strongly Heterogeneous Materials

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1. Introduction

In the past decades Multigrid methods have entered many fields of science as optimally efficient alternative to conventional numerical methods, showing huge computing time reductions as a result of which much larger and more realistic problems can be solved on small scale computers. Recently, Boffy et al. [1] showed the potential of multigrid methods for 3D heterogeneous material simulations. Unfortunately, the performance of the algorithm deteriorates with increasing property variations. In this paper it is shown how to overcome this limitation and benefit from the full potential of the efficiency of multigrid methods also for extremely heterogeneous materials. The resulting algorithm allows local property optimization in material design. Results are shown demonstrating the potential of the algorithm for different types of heterogeneous materials

2. Lamé's equations and Multigrid techniques

Linear elastic materials are assumed. In that case, given specific boundary conditions, the unknown displacements u, v and w in a 3-D domain must be solved from the 3-D Lamé's equation:

$$(\lambda u_{j,j})_{,i}+(\mu u_{i,j})_{,j}+(\mu u_{j,i})_{,j}=0 \quad i,j=1,2,3 \qquad (1)$$

In this equation, λ and μ denote the Lamé's coefficients, which may vary along all space directions.

3. Optimized intergrid operators

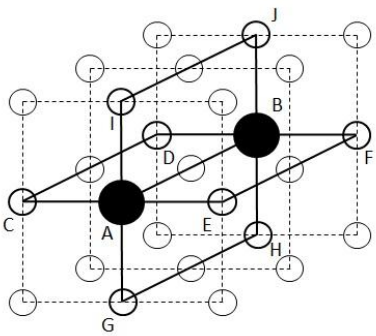

Fig1. Points involved in the definition of the coarse grid coefficient at the center of the link (AB)

Good multigrid performance requires accurate representation of the problem on coarser grids for convergence acceleration. For the 2-D problem this was shown by Alcouffe et al. [2]. Coarse grid coefficients and intergrid operators require special definitions. These coefficients have to accurately reflects the material behavior.Figure (1) shows the pathways used to fulfill an efficient coarse grid coefficient in the 3D case.

4. Simulations

Two examples are considered here. The first is for a Hertzian Pressure applied to an heterogeneous granular material with a void. Free boundaries are considered excepted at the bottom where zero displacements are imposed. Figure (2) shows the Von-Mises stress in the mid plan of the bulk. It shows both how the stress field is modified by the microstructure and the void. The second example is a fiber reinforced material subjected to a prescribed vertical displacement. The Young's modulus of the fibers is much larger than that of the bulk. The boundary conditions on the sides are free displacement. Figure (3) shows the tensile stress in the mid plan of the bulk. The pathways in the stress field are clearly highlighted. These interactions between the spheres allow to reduce local overstress. These results show how the developed model can be used in the design of new materials in engineering or tribology on a local scale.

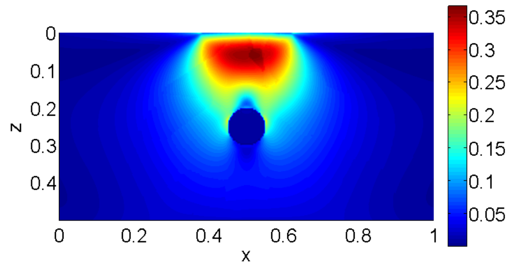

Fig2. Von-Mises stress : granular bulk and void

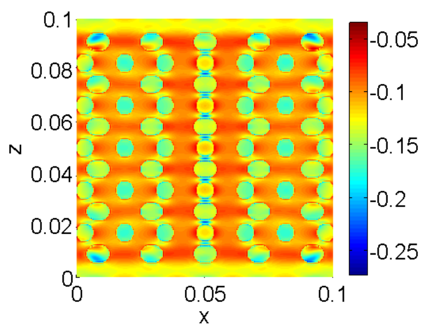

Fig3. Tensile stress : fiber reinforced material

5. References

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[2] Alcouffe,R. E.,Brandt,A.,Dendy, J. E. and Painter, J. W. "The Multigrid Method for the Diffusion Equation with Strongly Discontinuous Coefficients,"J.Sci.Stat.Comp., 2, 1981,430-454.